

Tuesday, 15 April

Weak vs Mass Eigenstates

$$-\mathcal{L}_{\text{ Yukawa }} = \sum_{m,n=1}^F \bar{U}_{m,L}^o \Gamma_{mn}^u \left(\frac{v + i\epsilon}{\sqrt{2}} \right) U_n^o R + (D, E, V) + \text{h.c.}$$

F = no of families

$$\bar{U}_L^o = \begin{pmatrix} u_L^o \\ c_L^o \\ t_L^o \end{pmatrix}$$

upper case - flavor vector
lower case - specific flavor

$$D_E = \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad E = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \quad V = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$-\mathcal{L}_{\text{ Yukawa }} = \bar{U}_L^o (M^u + h^u H(x)) U_R^o + (D, E, V) + \text{h.c.}$$

$$M^u = \Gamma^u \frac{v}{\sqrt{2}} = F \times F \text{ mass matrix}$$

$$h^u = \frac{M^u}{v} = \text{Yukawa coupling matrix}$$

Γ^u need not be diagonal

Hermitian
symmetric

To identify the mass eigenstates, we then perform a similarity transformation on the L & R fields

$$U_L^o \rightarrow S_L U_L^o = U_L \quad \Rightarrow M^u \rightarrow S_L^+ M^u S_R = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \equiv M_D^u$$

$$U_R^o \rightarrow S_R U_R^o = U_R$$

We can perform similar transformations for the

D, E, ν fields.

→ These are just Dirac mass terms. If ν are Majorana, or a mix of Dirac & Majorana, we have to extend the SM.

For the ν 's, their masses are $\ll m_e$, and for many purposes ≈ 0 .

→ The ν eigenstates are only really distinguished by their Weak interaction eigenstates, and usually expressed in that basis.

This is equivalent to choosing the same transformation as the "electrons"

$$S_L^\nu = S_L^E$$

The unitary transformation matrices can be determined by noting

$$M^\dagger M = \text{Hermitian}$$

$$S_L^{u\dagger} M_u^\dagger M_u + S_R^{u\dagger} = \begin{pmatrix} m_u^2 & 0 \\ 0 & m_c^2 \\ 0 & 0 \\ 0 & m_t^2 \end{pmatrix}$$

but this determination is not unique, as we could add an arbitrary phase to each flavor

$$S_L^u = \begin{pmatrix} e^{i\phi_u^L} & & \\ & e^{i\phi_c^L} & \\ & & e^{i\phi_t^L} \end{pmatrix} S_L^u$$

There is a lot of freedom in choosing these matrices.

The Standard prescription:

- pick arbitrary phases ϕ_f^L such that all unobservable phases in CKM matrix are rotated away

- we can then choose $\phi_f^R \geq 0, \epsilon \in \mathbb{R}$
 $\Rightarrow m_f^u \in \mathbb{R}, \geq 0$

\Rightarrow ~~equation~~

$$\begin{aligned} \Rightarrow \mathcal{L}_V &= \bar{\nu}_L i\cancel{D} \nu_L + \bar{\nu}_R i\cancel{D} \nu_R - \left[\bar{\nu}_L^0 M^u \bar{\nu}_R^0 \left(1 + \frac{H(v)}{v} \right) + h.c. \right] \\ &= \bar{\nu}_L S_L^{u+} S_L^u i\cancel{D} \nu_L + \bar{\nu}_R S_R^{u+} S_R^u i\cancel{D} \nu_R - \left[\bar{\nu}_L S_L^{u+} M^u S_L^u \bar{\nu}_R \left(1 + \frac{H}{v} \right) + h.c. \right] \\ &= \bar{\nu}_L i\cancel{D} \nu_L + \bar{\nu}_R i\cancel{D} \nu_R - \left[\bar{\nu}_L M_D^u \bar{\nu}_R + h.c. \right] \left(1 + \frac{H}{v} \right) \\ &= \sum_{f=1}^F \bar{\nu}_f i\cancel{D} \nu_f \\ &= \sum_{f=1}^F \bar{\nu}_f \left[i\cancel{D} - m_f \left(1 + \frac{H}{v} \right) \right] u_f \end{aligned}$$

and similarly for all fermions.

As we have discussed, there is nothing that requires the Yukawa & Weak eigenbasis to be simultaneously diagonal.

Recall, the charge current interaction couple

$$U - D, E - \nu$$

$$\mathcal{L} = \mathcal{L}_\psi - \frac{g}{2\sqrt{2}} \left(J_w^\mu W_\mu^- + J_w^{\mu+} W_\mu^+ \right) - \frac{gg'}{\sqrt{g'^2+g^2}} J_\alpha^\mu A_\mu - \frac{\sqrt{g'^2+g^2}}{2} J_z^\mu Z_\mu$$

$$\begin{aligned} J_w^\mu &= \sum_{m=1}^F \left[\bar{e}_m^0 \gamma^\mu (1-\gamma_5) v_m^0 + \bar{d}_m^0 \gamma^\mu (1-\gamma_5) u_m^0 \right] \\ &= \bar{E}^0 \gamma^\mu (1-\gamma_5) v^0 + \bar{D}^0 \gamma^\mu (1-\gamma_5) u^0 \\ &= 2 \bar{E}_L \gamma^\mu S_L^u S_L^{v\dagger} v_L + 2 \bar{D}_L \gamma^\mu S_L^d S_L^{u\dagger} u_L \end{aligned}$$

$$V_L^\dagger = S_L^e S_L^{v\dagger} = (S_L^{v\dagger} S_L^e)^\dagger$$

$$V_q = S_L^{u\dagger} S_L^d$$

for the quarks

$$V_q = V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

These matrices are arbitrary ~~complex~~^{unitary} matrices, with

$$(V_q^\dagger V_q)_{nm} = \delta_{nm}$$

$\Rightarrow F^2$ real parameters.

We can use the arbitrary ϕ_L phases to remove

$$2F-1$$

phase differences

$$\Rightarrow F^2 - 2F + 1 = \frac{F(F-1)}{2} + \frac{(F-1)(F-2)}{2}$$

angles + phases

It is standard to chose the upper weak flavor to correspond to the mass eigenstates, with arbitrary mixtures in the lower component.

$$D_w^0 = V D_M$$

We notice of course that with 2 generations, we only need a single mixing angle to describe the rotation

whereas with 3 generations ($F=3$) we need two mixing angles and one phase.

The standard parameterization

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{23} & S_{23} \\ 0 & -S_{23} & C_{23} \end{pmatrix} \begin{pmatrix} C_{13} & 0 & S_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -S_{13} e^{i\delta} & 0 & C_{13} \end{pmatrix} \begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S_{ij} = \sin \theta_{ij}$$

$$C_{ij} = \cos \theta_{ij}$$

The phase, δ , gives rise to cp-violation: 3 families are a necessary condition for CP violation

In magnitude, the CKM elements are

$$|V_{CKM}| \sim \begin{pmatrix} 0.9742 & 0.226 & 0.0036 \\ 0.226 & 0.973 & 0.047 \\ 0.0087 & 0.041 & 0.9991 \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

The off diagonal elements allow the heavier quarks to decay to lighter quarks, which is why nature only has stable (u,d) matter.

Unitarity constraints the matrix elements

$$V^+ V = \underline{1}$$

$$\Rightarrow |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Searching for violations of this unitarity is one of the principle means we look for BSM physics.

e.g. a 4th generation of quark would add a new row/column to V_{CKM} and therefore an apparent violation of unitarity if only considering 3 of the 4 flavors.

Presently, there is some tension, but no clear signs of unitarity violation in CKM matrix.

These constraints are often represented with a triangle

$$\begin{array}{c} V_{ub}^* V_{ud} \\ \times \\ V_{cb}^* V_{cd} \\ \times \\ V_{tb}^* V_{td} \end{array}$$

$$\Rightarrow \begin{array}{ccc} \bar{\rho} + i\bar{\eta} & & 1 - \bar{\rho} - i\bar{\eta} \\ & \swarrow & \searrow \\ (0,0) & -1 & (1,0) \end{array}$$

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

It is common & useful to parameterize CKM with Wolfenstein parameterization

$$V_q = \begin{pmatrix} 1 - \bar{\rho}/2 & \bar{\rho} & A\bar{\lambda}^3(\bar{\rho} - i\bar{\eta}) \\ -\bar{\lambda} & 1 - \bar{\lambda}^2/2 & A\bar{\lambda}^2 \\ A\bar{\lambda}^3(1 - \bar{\rho} - i\bar{\eta}) & -A\bar{\lambda}^2 & 1 \end{pmatrix}$$

Brief Aside on neutral current interactions

recall the neutral current interaction is diagonal in flavor

$$\mathcal{L}_{NC} = g_e \bar{e} \gamma^\mu A_\mu e - \frac{g}{2\cos\theta} \bar{\nu}_L \gamma^\mu Z_\mu \nu_L$$

$$- \sqrt{\frac{G_F M_Z^2}{2\sqrt{2}}} \left[(4\sin^2\theta_W - 1) \bar{e} \gamma^\mu e + \bar{e} \gamma^\mu \gamma_5 e \right] Z_\mu$$

+ similar term for quarks.

When only the u,d,s quark flavors were known, this led to conceptual problems

The neutral current Weak interactions w/ just these three quarks led to predictions of both

$$\Delta S=0$$

$$\Delta S=1$$

neutral current interactions

$$\bar{u} W u + \bar{d}' W d'$$

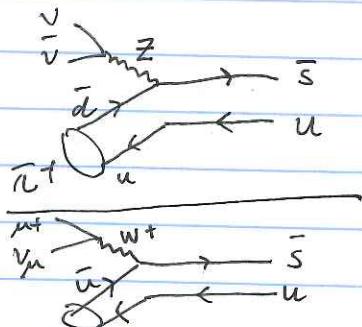
$$d' = d \cos\theta_W + s \sin\theta_W$$

$$\Rightarrow \Delta S=0: \bar{u}u + \bar{d}d \cos^2\theta_C + \bar{s}s \sin^2\theta_C$$

$$\Delta S=1: (3d + 2s) \cos\theta_C \sin\theta_C$$

implying strange hadrons should decay via neutral current interactions. However, it was observed

$$\frac{K^+ \xrightarrow{NC} \pi^+ \nu \bar{\nu}}{K^+ \xrightarrow{CC} \pi^0 \mu^+ \nu_\mu} \approx \frac{1.7 \times 10^{-10}}{3.4 \times 10^{-2}}$$



These Flavor Changing Neutral Currents (FCNC) were observed to be very suppressed.

- In fact, FCNC are one of the main means of constraining BSM models.

Glashow, Eliopoulos & Maiani noticed the inclusion of a 4th quark, g could naturally explain this suppression.

$$\Psi' = -\sin\theta_c d + \cos\theta_c s$$

$$\Rightarrow \Delta S=0: \bar{u}u + \bar{c}c + (\bar{d}d + \bar{s}s) \cos^2\theta_c + (\bar{s}s + \bar{d}d) \sin^2\theta_c \\ = \bar{u}u + \bar{c}c + \bar{d}d + \bar{s}s$$

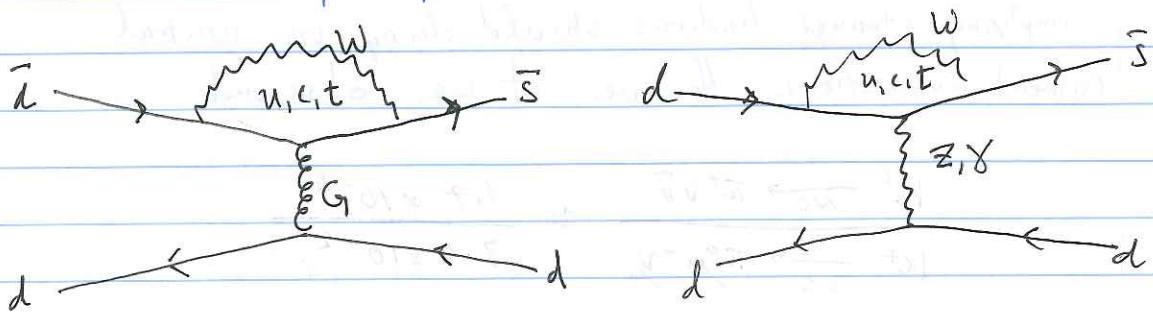
$$\Delta S=1: (\bar{s}d + \bar{d}s - \bar{s}d - \bar{d}s) \cos\theta_c \sin\theta_c = 0.$$

This is known as the GIM mechanism. (1970)

Bjorken & Glashow speculated the existence of c in 1964

GIM requires the existence of c .

The LO Weak interactions prohibit FCNC, but radiative corrections allow for them.



Consequences of Weak CP violation: neutral Kaon system.

The K appear in isospin doublets

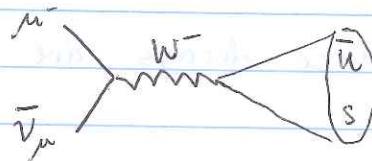
$$K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} = \begin{pmatrix} \bar{s}u \\ \bar{s}d \end{pmatrix} \quad \bar{K} = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix} = \begin{pmatrix} \bar{d}s \\ \bar{u}s \end{pmatrix}$$

The Kaons decay weakly.

Also, the charge neutral Kaons can mix.

Strangeness is not conserved by H_W .

The simplest decays are leptonic



Recall the π decay. The Kaon is just the heavy cousin of the π , so the decay amplitude will be very similar.

$$iA \propto iG_F f_K q^\mu \bar{u}_{\mu\alpha} \gamma_\mu (1 - \gamma_5) v_{(5)}$$

The decay constant characterizes the kaon wave function at the origin

$$m_K f_K^2 \sim |\psi_K|_0^2$$

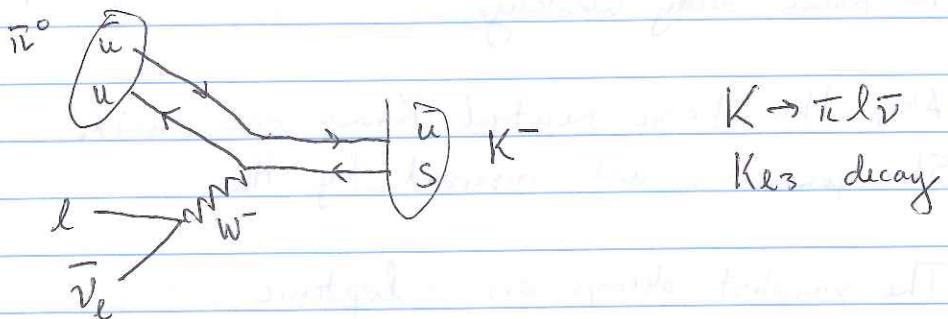
i.e. it is the probability both the quark and anti-quark will be at the same point, and thus able to annihilate.

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Just as ω/π , the helicity structure leads to a dominant rate for μ vs e

$$\frac{\Gamma(K \rightarrow e\bar{\nu})}{\Gamma(K \rightarrow \mu\bar{\nu})} \approx \frac{1.584 \times 10^{-5}}{0.6355} \sim 2.5 \times 10^{-5}$$

The kaon can also decay semi-leptonically



The branching ratio for these decays are very similar

$$\Gamma_{K_{ls}} = 0.0507$$

$$\Gamma_{K_{ls}} = 0.0335$$

In this decay, there is no spin/angular momentum miss match, since the π and W can be in a relative p-wave, and thus the lepton mass does not play a significant role in the decay.

Contrast with leptonic decay

(π or K) is spin-0

$\Rightarrow l \bar{\nu}$ must have the same helicity to conserve angular momentum

$$\bar{\nu}_e \leftarrow \pi^+ \rightarrow l^-$$

but the Weak interactions couple to left-handed leptons
right-handed anti-leptons

$\frac{1}{2} p_+$

\Rightarrow the only way to have the helicities the same is through a "spin-flip" with the lepton mass - the mass means the helicity is not a good QM number since a boost can change helicity

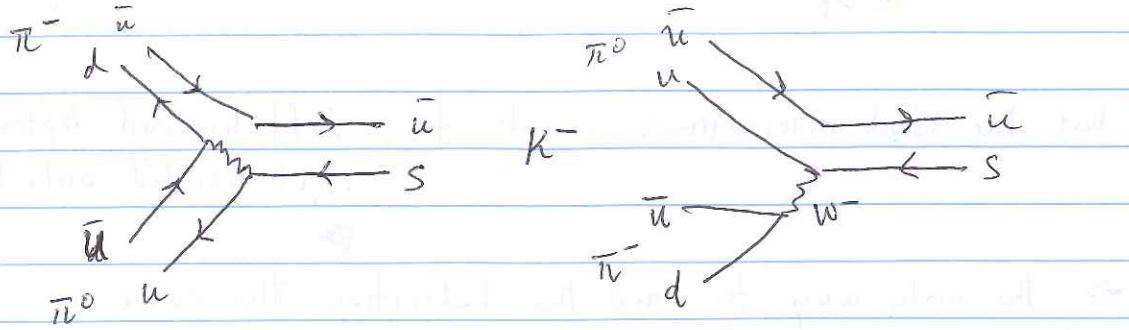
\Rightarrow or you can think the mass of the lepton is an interaction which allows for helicity flip,

and so the only way for the decay to proceed is through the "mass" interaction which provides an overlap of the two different helicity states of the lepton.

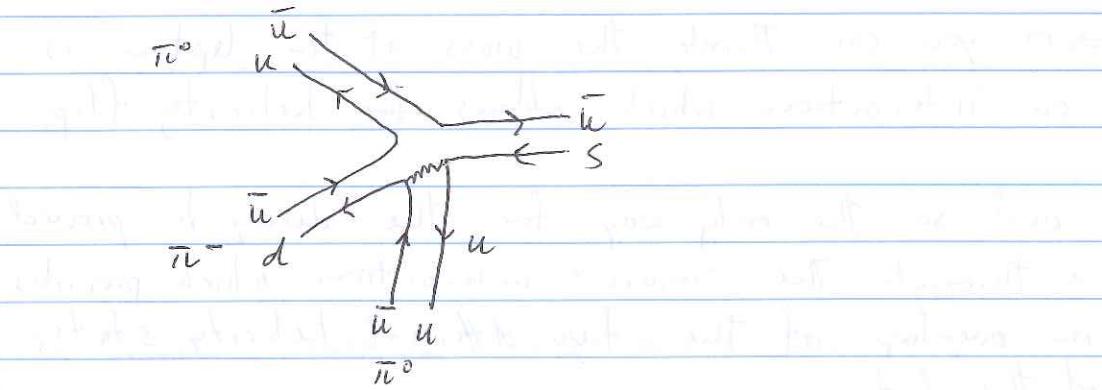
\Rightarrow if the leptons were all massless, the π would not decay.

Back to our semi-leptonic K decay, the relative momentum between π -W alleviates this problem, and the lighter m_e allows for more phase space leading to the slight enhancement.

The Kaon can also Weak decay to purely hadrons



There are also 3π decay channels



$$\Gamma(K^\pm \rightarrow \pi^\pm \pi^0) = 0.2066$$

$$\Gamma(K^\pm \rightarrow \pi^\pm \pi^+ \pi^-) = 0.0559$$

$$\Gamma(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0) = 0.0176$$

The 2π decay mode dominates from phase space

The charged 3π decay is ~ 3 times more probable.
This is understood in terms of the dominance of
the " $\Delta I = \frac{1}{2}$ rule" in weak decays